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When to Copy or Avoid an Opponent's Strategy

STEVEN A. FRANK

Department of Ecology and Evolutionary Biology, University of California, Irvine, CA 92717, *U.S.A.*

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Models of two-player games are analyzed in which contestants strive to maximize relative success (market share). Each contestant divides its resources among a set of investment strategies. For a particular investment strategy the contestants may receive different expected rates of return. Each strategy also returns to both contestants an additional payoff that is unpredictable. Depending on particular assumptions, a contestant may maximize relative success by copying or avoiding its opponent's allocation pattern. In other cases a contestant may be favored to diversify its investments equally among strategies and minimize its total variance in returns, or to invest only in one strategy and maximize its total variance in returns.

Introduction

Suppose that, between two contestants, the winner is the one with the most resources at the end of a specified period. The battle for greatest *relative* success proceeds as each contestant continually accrues resources in return for its investment in one or more of the K available strategies. When does it pay for one contestant to copy or to avoid the strategies followed by its opponent?

A simple example illustrates how copying or avoiding an opponent may be advantageous. Let each of the K strategies have an expected rate of return that depends on the amount of resource invested. The actual returns obtained in any time period are unpredictable because of environmental uncertainty or difficulty in assessing success rate. Finally, assume that contestant A_1 initially has more resources than contestant A_2.

If A_1 can distribute its resources exactly as A_2, then it is guaranteed to maintain its relative advantage by the current amount. On the other hand, if A_2 can avoid the strategies followed by A_1, it has a chance to get luckier than A_1 in its payoffs and gain the lead in resources.

In this paper I analyze how copying or avoiding an opponent's behavioral or economic strategy influences relative success. Most of the necessary mathematical results can be obtained by a new interpretation of Gillespie's (1973) analysis of gene frequency evolution with varying selection coefficients.

Definitions

Let the two contestants, A_1 and A_2, at time t have fractions X_t and $(1-X_t)$ of the total resource pool to invest in their future growth. In each time step there is a

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period of growth or resource accumulation in which each contestant grows independently from the other according to its returns from a variety of investment strategies. Before the start of the next time step there is a competition in which the total resources of the two contestants are normalized to a constant level uncorrelated with the independent growth and development of each contestant.

For the period of independent development, define the rate of growth or increase in competitive potential for A_1 as $1+U_t$, where:

$$U_t = m_u + \sum_{i=1}^{K} p_i \delta_{it},$$

where the expected value of U_t is m_u, the fraction of its resources that A_1 invests in the ith investment strategy is p_i, and δ_{it} is a random variable with mean zero, variance γ, and covariance between (δ_i, δ_j) equal to zero when i and j are different. Likewise for A_2, growth is $1+V_t$, with:

$$V_t = m_v + \sum_{i=1}^{K} q_i \delta_{it},$$

where the expected value of V_t is m_v, and the fraction of its resources that A_2 invests in the ith investment strategy is q_i.

Assuming that each contestant's allocation pattern is constant over time, the variances in success and covariance in success between opponents can be written as:

$$\sigma_u^2 = \gamma \rho_u = \gamma \sum_{i=1}^{K} p_i^2,$$

$$\sigma_v^2 = \gamma \rho_v = \gamma \sum_{i=1}^{K} q_i^2,$$

$$\sigma_{uv} = \gamma \rho_{uv} = \gamma \sum_{i=1}^{K} p_i q_i,$$

where ρ_u, ρ_v and ρ_{uv} are the average correlations in returns between randomly chosen units of investment for, respectively, pairs of A_1, pairs of A_2, and pairs of A_1 and A_2. A contestant's variance in success for each time step is minimized by spreading its resources equally among all strategies, and the covariance between competitors increases as the investment patterns become more similar for the two contestants.

The trajectory for relative success of A_1 is:

$$\Delta X = \frac{X_t(1-X_t)(U_t - V_t)}{1 + X_t U_t + (1-X_t) V_t},$$

which is Gillespie's [1973: eqn (1)] with his notation. The effects of correlations between the strategies of opponents occur through the variances of U and V and the covariance between them. We can therefore apply Gillespie's analysis for the dynamics of X as a function of the means, variances and covariances of U and V to answer questions about the optimal copying or avoiding of an opponent's strategy.

When to Copy or Avoid an Opponent

I will analyze two different measures of success: (1) the probability that one contestant is the first to drive its opponent to near extinction, and (2) the probability that one contestant has more than 50% of all resources at the end of a fixed time interval. Predictions about optimal strategy depend on which measure of success is used and on the information available about an opponent's allocation pattern, relative resource level, and expected returns m.

NEAR MONOPOLY

How should one contestant distribute its resources in order to maximize its probability of being the first to have nearly all of the available resources? In population genetics this is the problem of first passage into quasi-fixation. From Gillespie (1973) and the above definitions, the probability of near monopoly for A_1 increases as the following ratio increases:

$$W = \frac{2\Delta\tilde{m} + \rho_v - \rho_u}{\rho_u + \rho_v - 2\rho_{uv}}, \tag{1}$$

where $\Delta\tilde{m} = (m_u - m_v)/\gamma$. The numerator is proportional to the difference in geometric mean success over time and the denominator is proportional to the variance in changing resource levels caused by stochastic fluctuations in payoff for each strategy. When the denominator is zero, the result is determined entirely by the sign of $\Delta\tilde{m}$ which, when positive, implies that A_1 will dominate with certainty and when negative implies that A_2 will dominate with certainty. In general, there are four cases depending on information available about $\Delta\tilde{m}$ and an opponent's allocation pattern. When there is no information about $\Delta\tilde{m}$, I assume that each contestant behaves as if $\Delta\tilde{m}$ had a probability distribution that is symmetric about zero which, as it turns out in the following cases, is equivalent to assuming $\Delta\tilde{m} = 0$.

(a) No information about $\Delta\tilde{m}$ or opponent's allocation pattern. The only stable strategy is to spread one's resources evenly across all K options. To see this, suppose that both A_1 and A_2 adopt this strategy, which yields $\rho_u = \rho_v = 1/K$ and no expected evolutionary change in relative frequency. Since $1/K$ is the minimum possible value for ρ, any other strategy followed by A_2 causes the condition in eqn (1) to be positive and thus to favor A_1, with the same argument applying to any deviation in strategy by A_1.

(b) Information about an opponent's allocation pattern but not about $\Delta\tilde{m}$. Suppose that one contestant can adjust its allocation of resources according to the fixed allocation pattern of its opponent. I will show that, under certain assumptions, the best response is to copy the opponent's allocation pattern, with the exception that a small investment should be made in a strategy ignored by the opponent. To analyze this situation I first rewrite eqn (1) as:

$$W = \frac{\sum q_i^2 - \sum p_i^2}{\sum p_i^2 + \sum q_i^2 - 2\sum p_i q_i} = \frac{\sum (q_i - p_i)(q_i + p_i)}{\sum (q_i - p_i)^2},$$

where p_i and q_i are the proportion of resources that A_1 and A_2 respectively invest in the ith strategy, and all sums are over $i = 1$ to K.

Suppose that A_2s allocation of resource among strategies is fixed and known to A_1 and that there is at least one strategy in which A_2 invests nothing, $q_1 = 0$. Let A_1 be free to distribute resources under the constraints that $p_1 = \varepsilon$, and for $i > 1$, $p_i = q_i - \varepsilon/(K-1)$. A_1 increases the extent to which it copies A_2 by decreasing ε. When $\varepsilon = 0$ the allocation patterns are identical and there is clearly no change over time in proportion of resource. If A_1 avoids A_2 as much as possible, $\varepsilon = 1$, then $W \leq 0$ and A_2 is more likely than A_1 to be the first to approach near monopoly. When ε is small, W increases as $\varepsilon \to 0$, suggesting that A_1 does best by matching A_2s allocation pattern except for a small investment in a strategy ignored by A_2. Clearly, if opponents have simultaneous knowledge about their competitor and can adjust their allocation accordingly, each can do no better than spread resources equally among the K strategies.

(c) Information on $\Delta\tilde{m}$ but no information about an opponent's allocation pattern. If A_1 has a higher expected return per unit investment, $\Delta\tilde{m} > 0$, then it does best by spreading its resources equally among the K strategies. This amounts to playing it safe when having a steady advantage. The formal argument is similar to that in (a).

When A_1 has a lower expected return, $\Delta\tilde{m} < 0$, then it does best by investing all of its resources into one strategy, thereby maximizing its chance to get a run of good luck and overtake A_2 in spite of its disadvantage. This result is obtained by noting that A_2 will spread its resources equally among the K strategies, thus the derivative of W with respect to ρ_u is $-2\Delta\tilde{m}/(\rho_u - K^{-1})^2$. Since $\Delta\tilde{m}$ is negative, W increases with ρ_u, and W is therefore a maximum at $\rho_u = 1$.

(d) Information on both $\Delta\tilde{m}$ and the opponent's allocation pattern. If a contestant has a higher expected return, then it does best by exactly copying its opponent because this will lead to a steady deterministic increase in resource share. If a contestant has a lower expected return, then it does best by avoiding its opponent as much as possible, since this maximizes the probability of a run of good luck as in the previous case.

MAJORITY RULE

The goal in this case is to have the majority of resource at a fixed time in the future. For example, two neighboring ant colonies may face an inevitable war later in the year. Success in the war may depend on relative colony size, which in turn depends on success in foraging for a variety of nutrients.

From Gillespie (1973) the probability of controlling more than one-half of the available resources after n periods of competition is:

$$\text{Prob}\,(X_n > 0{\cdot}5) = 1 - Z(W, \infty),$$

where Z is the probability of a standard normal between W and ∞, and W is:

$$W=\frac{n[(m_u-m_v)+(1/2)\gamma(\rho_v-\rho_u)]+\ln[x_0/(1-x_0)]}{\sqrt{n\gamma(\rho_u+\rho_v-2\rho_{uv})}}.$$

The goal of A_1 is therefore to maximize W.

I will analyze the case in which expected returns are the same for each contestant, $m_u=m_v$, and the contestants have no knowledge of an opponent's allocation pattern but do have knowledge of relative resource share at the start of the contest, $t=0$. This case is sufficient to illustrate the qualitative patterns that occur, which depend mainly on n and the initial share of resource.

When $x_0=0{\cdot}5$, then both contestants are favored to spread resources equally among all K strategies, $\rho_u=\rho_v=\rho_{uv}=1/K$. This can be derived by the arguments described under case (a) in the previous section. When $x_0>0{\cdot}5$, inspection of the expression for W shows that W is maximized when resources are spread equally among all strategies, $\rho_u=\rho_{uv}=1/K$, for any allocation pattern followed by A_2 that is unknown by A_1. This same investment pattern maximizes the probability that A_1 can maintain its lead.

When $x_0<0{\cdot}5$, contestant A_2 will be expected to adopt an equal allocation of resources among the K options, and contestant A_1 must balance taking a risk to increase its variance in success by investing in only a few options vs. playing it safe to maximize its expected rate of long-term growth by investing evenly among options. The problem of maximizing W is now equivalent to minimizing the following expression with respect to ρ_u:

$$Y=(n\gamma/2)\sqrt{\rho_u-(1/K)}+\frac{C}{\sqrt{\rho_u-(1/K)}},$$

where $C=-\ln[x_0/(1-x_0)]$. Solving yields:

$$\begin{aligned}\rho_u&=\frac{1}{K}+\frac{2C}{n\gamma}, &\qquad C&<\frac{n\gamma(K-1)}{2K},\\ &=1, &\qquad C&\geq\frac{n\gamma(K-1)}{2K}.\end{aligned}$$

The optimal strategy weighs the initial disadvantage C against the length of the contest n: the greater the initial disadvantage, the more concentrated the investment into one or a few strategies, whereas the longer the contest, the more evenly distributed the investment pattern. This implies that a risky strategy only works well over a short time period.

Discussion

The optimum pattern for allocating resources depends on the measure of success. When contestants strive to be the first to control nearly all of the available resources, maximum diversification of resources among strategies is favored when neither contestant has information about the other's allocation pattern or expected rate of return on investment. Diversification lowers the variance in success in each time step and therefore maximizes the geometric rate of growth. This supports Real's (1980) conclusion that diversification is typically favored. By contrast, when a contestant knows that it is at a disadvantage in expected rate of returns per unit investment, its best chance of being the first to dominate is to invest all its resources in a single strategy. This maximizes the variance in success and thus the variance in the time trajectory of relative control of resources, which in turn maximizes the chance that the inferior competitor will get lucky and win the contest. (Other interesting game-theory examples in which a weaker contestant is favored to play a riskier strategy can be found in Dresher, 1981: chapters 9 and 10.) Finally, when one contestant can adjust its allocation pattern according to the pattern of its opponent, it is often most advantageous to match the opponent's pattern except for making a small investment in the strategy in which the opponent has invested least.

When contestants strive to control the majority of available resources after a fixed interval, the initial share and the length of the contest are critical determinants of behavior. If the contest is very long, maximum diversification of behavior is typically favored. If the contest is short, as it may be in behavioral dominance interactions, the initially weaker contestant is favored to take a risk and invest heavily in one strategy, thus maximizing its chance of hitting a big payoff and overtaking its opponent. The initially stronger contestant is favored to diversify and safely protect its lead.

These general results apply to any two-player game in which there is uncertainty in payoff and the goal is to increase relative success (market share). Behavioral examples include competing colonies of social insects or groups of co-operatively breeding birds, where the contested resource is territory and the strategies are various foraging options. Success in foraging may determine rate of colony growth, which in turn may determine territorial dominance. If the neighbors will soon go to war, then holding a majority in territory and colony size may be critical. The initially smaller colony may be favored to gamble on a narrow allocation pattern, whereas the initially stronger colony may be favored to diversify its resource gathering strategies and minimize chance fluctuations in success.

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